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B. E. (Third Semester) Examination, April-May 2020

(New Scheme)

(IT Engg. Branch)

DISCRETE STRUCTURES

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : *Answers all questions. Part (a) is compulsory and carries 2 marks. Answer any* ***two*** *parts from (b), (c) and (d) and carries 7 marks each.*

Unit-I

1. (a) Define Tautology. 2

(b) Let $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$ be the set of all positive divisors of 30. If

$a \vee b$ = L.C.M. of a and b,

$a \wedge b$ = H.C.F. of a and b, $a' = 30/a$,

then prove that the algebraic structure $(B, \vee, \wedge, ')$ is a Boolean algebra. 7

(c) Define disjunctive normal form and change the following Boolean function to disjunctive normal form :

$$f(x, y, z) = \left[x + (x' + y)'\right] \cdot \left[x + (y' \cdot z')'\right]$$ 7

(d) Draw a circuit for the following Boolean function and replace it by a simple one :

$$F(x, y, z) = x \cdot z + \left[y \cdot (y' + z) \cdot (x' + x \cdot z')\right]$$ 7

Unit-II

2. (a) Define power set with a example. 2

(b) If I is the set of non-zero integers and a relation R

is defined by xRy if $x^y = y^x$, where $x, y \in I$, then. Is the relation R on equivalence relation. 7

(c) Define partially ordered set. Let S be any class of sets. Prove that the relation of set inclusion "$\subseteq$" is a partial order relation on S. 7

(d) If Q is the set of rational numbers and $f : Q \to Q$ is defined by $f(x) = 3x + 2, x \in Q$, then prove that the f is one-one and onto. Find also f^{-1}. 7

Unit-III

3. (a) Show that there is no solution of :

$$25x \equiv 12 \pmod{10}$$ 2

(b) Show that the set of all integers I forms a group with respect to the binary operations "*" defined by the rule $a * b = a + b + 1, \forall\ a, b \in I$. 7

(c) If G be the multiplicative group of set $\{1, -1\}$ and G' be the additive group of residue classes modulo

2 i.e. $G' = (\{0, 1\}, +_2)$, then show that these are isomorphic group. 7

(d) If the system $(R, +, \cdot)$ be a ring R, then prove that : 7

(i) $a.0 = 0.a = 0, \forall a \in R$

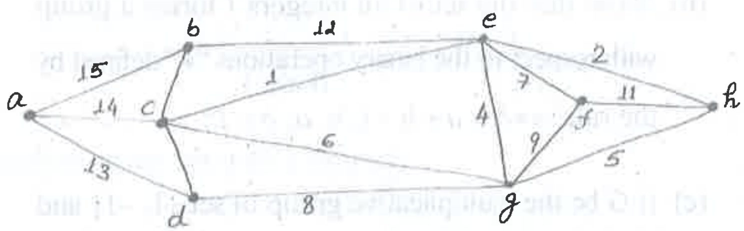
(ii) $a.(b-c) = a.b - a.c, \forall\ a, b, c \in R$

Unit-IV

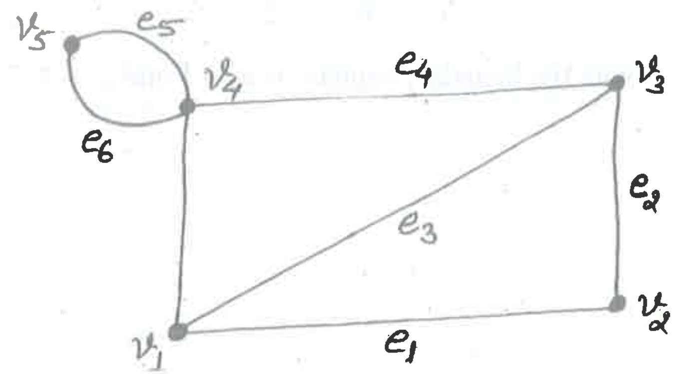
4. (a) Define self-loop in a graph. 2

(b) Prove that, the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$ 7

(c) Using Kruskal's algorithm, find a minimum spanning tree for the graph : 7



(d) Define incidence matrix and adjacency matrix. Write the incidence matrix of the graph shown below : 7



Unit-V

5. (a) Find n if $2^nP_2 + 50 = {}^{2n}P_2$. 2

(b) Show that :

$$1^2 + 2^2 + \ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \geq 1$$

by mathematical induction. 7

(c) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. 7

(d) Solve by the method of generating functions the recurrence relation

$$a_r - 5a_{r-1} + 6a_{r-2} = 2,\ r \geq 2$$

with the boundary conditions $a_0 = 1$ and $a_1 = 2$. 7